1999–2000 CATS ASSESSMENT
Open-Response Item Scoring Worksheet

Grade 11 – Mathematics

The **academic expectations** addressed by the open-response item "Rope Strength" are:

1.5-1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.11 Students understand mathematical change concepts and use them appropriately and accurately.

The **core content** addressed by this item includes:

MA-H-4.3.2 Algebra and Related Topics (Relationships): Students will understand how formulas, tables, graphs, and equations of functions relate to each other.

MA-H-4.3.5 Algebra and Related Topics (Relationships): Students will show how equations and graphs are models of the relationship between two real-world quantities (e.g., the relationship between degrees Celsius and degrees Fahrenheit).

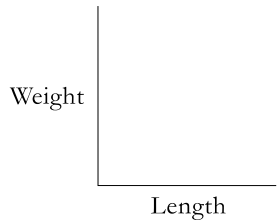
MA-H-4.1.4 Algebra and Related Topics (Concepts): Students will identify linear, quadratic, absolute value, and exponential functions from graphs and equations.

1999–2000 CATS ASSESSMENT
Open-Response Item Scoring Worksheet

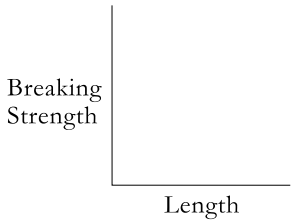
Rope Strength

Manufacturers consider several different features when making rope for various uses. In this problem, you will graph or find a relationship between these different features. (To answer this question, you will need to draw 3 graphs in your Student Response Booklet.)

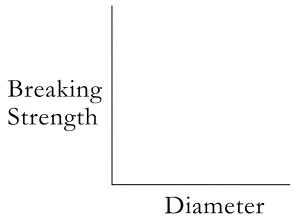
a. In your Student Response Booklet, copy the labeled axes below and sketch a graph showing that if you double the length of a piece of rope, you double its weight.



b. In your Student Response Booklet, copy the labeled axes below and sketch a graph showing that the breaking strength of a rope is not affected by its length.



c. In your Student Response Booklet, copy the labeled axes below and sketch a graph showing that if you double the diameter of a rope, you multiply its breaking strength by 4.



d. The table of data below shows the relationship between the diameter of a rope and its breaking strength. Using the data, write a formula for the breaking strength (S) in terms of the diameter (d).

diameter (in mm)	0	2	4	6	8	10	12
approximate breaking strength (in kg)	0	80	320	720	1280	2000	2880

BE SURE TO LABEL YOUR ANSWERS (a), (b), (c), AND (d).

SCORING GUIDE
Grade 11 Mathematics

Score	Description
4	Student scores 4 points.
3	Student scores 3 to 3.5 points.
2	Student scores 1.5 to 2.5 points.
1	Student scores .5 to 1 point. **OR** Student shows some understanding for determining graphs or writing a formula.
0	Response is totally incorrect or irrelevant.
Blank	No response.

Note: At the 4 level student must have both axes correctly labeled.

SCORE POINTS

part a

1 point — Any line that intersects the origin for x>0.

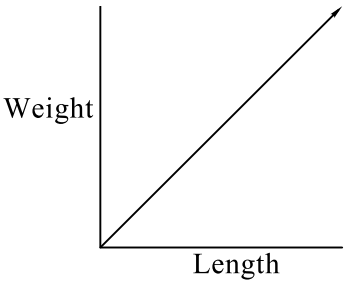
OR

.5 point — Any line that intersects the origin but the domain is not restricted to x>0.

OR

.5 point — Graph shown as a discrete function with correct shape (need at least 3 points to determine shape).

e.g.



part b

1 point — Linear graph (parallel to horizontal axis) which shows that breaking strength is not affected by length.

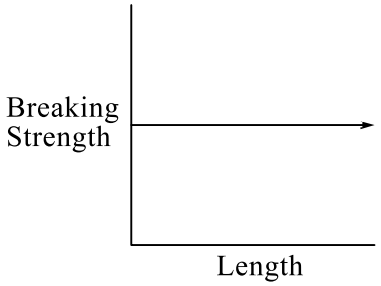
OR

.5 point — Linear graph (parallel to horizontal axis) but the domain is not restricted to x>0.

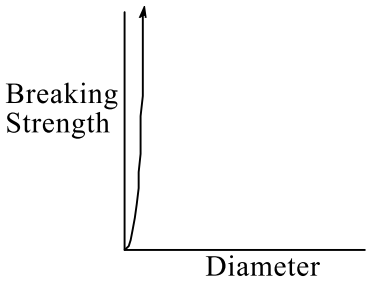
OR

.5 point — Graph shown as a discrete function with correct shape (need at least 3 points to determine shape).

e.g.



SCORING GUIDE
Grade 11 Mathematics

part c	1 point	Any parabola centered on the origin for x>0.	e.g. 
	OR		
	.5 point	Parabolic graph, but the domain is not restricted to x>0.	
	OR		
	.5 point	Graph shown as a discrete function with correct shape (need at least 3 points to determine shape).	
part d	1 point	correct formula ($S=20d^2$)	
	OR		
	.5 point	correct expression or equivalent expression	

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

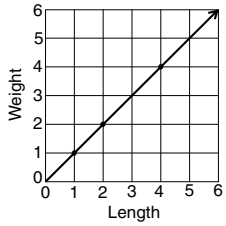
Sample Student Response Scored a 4

Student Response

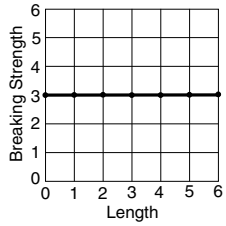
In the manufacture of rope, a company must learn about all relationships of the different aspects of each type of rope. Below are shown several graphs indicating these relationships:

A. Weight\Length comparisons in rope

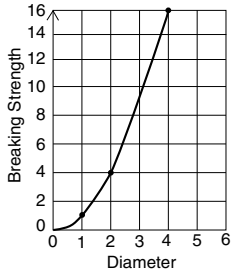
This graph shows that the weight doubles with the length, showing linear progression.



B. This graph shows no relationship between the length and breaking strength.



C. This graph shows the breaking strength multiplied by 4 when the rope diameter is doubled.



D. A formula determining the relationship between the diameter (d) and breaking strength can be

breaking strength = $d \times (d/2)40$

Student draws a correct graph showing a line that passes through the origin. Labels shown are correct. (1 point)

Student draws a correct graph showing a line that is parallel to the *x*-axis. Labels shown are correct. (1 point)

Student draws a correct graph showing a parabola (curve) centered on the origin. Labels shown are correct. (1 point)

Student states a correct formula (i.e., gives an equivalent formula to that shown in the scoring guide). (1 point)

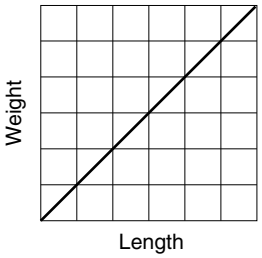
Overall, the student earns 4 points, demonstrating a strong understanding of the mathematical concept of change and a strong ability to model change, both graphically and algebraically, in two-variable situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

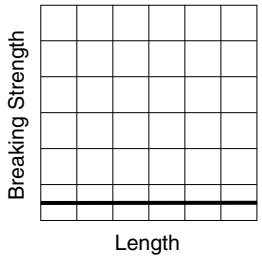
Sample Student Response Scored a 4

Student Response

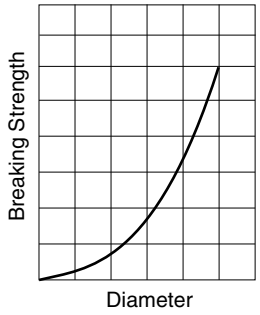
a.



b.



c.



d. $S = d \bullet d \bullet 20$

$S = d^2 \bullet 20$

← Student draws a correct graph showing a line that passes through the origin. Labels shown are correct. (1 point)

← Student draws a correct graph showing a line that is parallel to the *x*-axis. Labels shown are correct. (1 point)

← Student draws a correct graph showing a parabola (curve) centered on the origin. Labels shown are correct. (1 point)

← Student states the correct formula. (1 point)

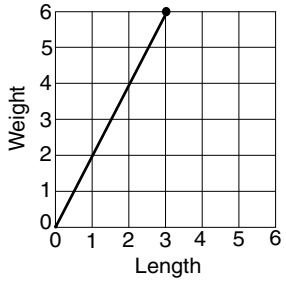
Overall, the student earns 4 points, demonstrating a strong understanding of the mathematical concept of change and a strong ability to model change, both graphically and algebraically, in two-variable situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

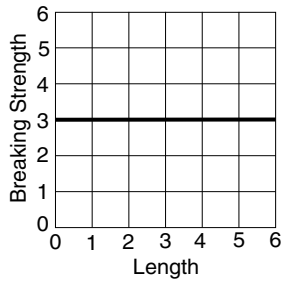
Sample Student Response Scored a 3

Student Response

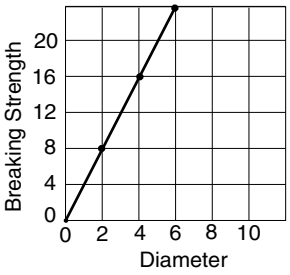
a.



B.



C.



d. formula $(d\,(20))d = S$

Expl. If you Multiply your diameter by 20 and the multiply your answer by your diameter you get your breaking strength.

← Student draws a correct graph showing a line that passes through the origin. Labels shown are correct. (1 point)

← Student draws a correct graph showing a line that is parallel to the *x*-axis. Labels shown are correct. (1 point)

← Student draws an incorrect graph, indicating a linear relationship between diameter and breaking strength. (0 points)

← Student states a correct formula (i.e., gives an equivalent formula to that shown in the scoring guide). (1 point)

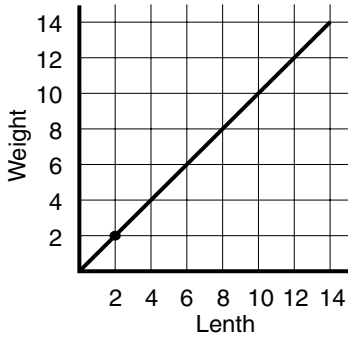
Overall, the student earns 3 points, demonstrating a general understanding of the mathematical concept of change and a general ability to model change, both graphically and algebraically, in two-variable situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

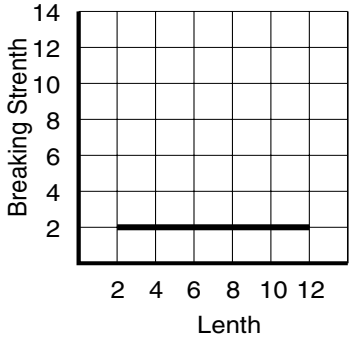
Sample Student Response Scored a 2

Student Response

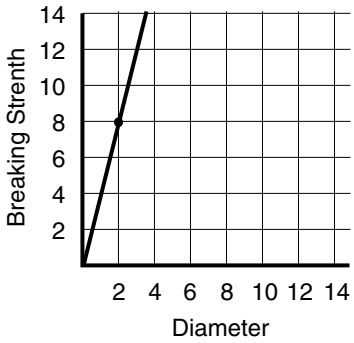
a



b)



c)



d) Can't remember

← Student draws a correct graph showing a line that passes through the origin. Labels shown are correct (even though one is misspelled). (1 point)

← Student draws a correct graph showing a line that is parallel to the *x*-axis. Labels shown are correct (even though they are misspelled). (1 point)

← Student draws an incorrect graph, indicating a linear relationship between diameter and breaking strength. (0 points)

← Student does not state a formula. (0 points)

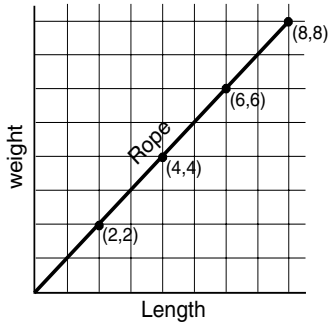
Overall, the student earns 2 points, demonstrating some understanding of the mathematical concept of change and some ability to model change graphically in two-variable situations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

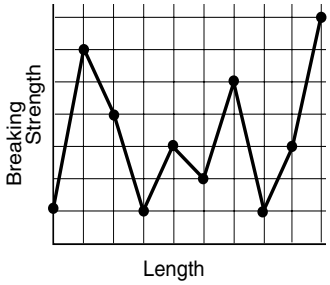
Sample Student Response Scored a 1

Student Response

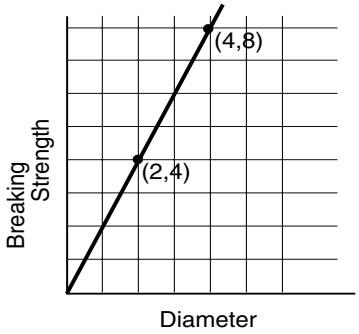
a.



b.



c.



d.

	d		s
$\frac{1}{20}$	2×40	=	80
$\frac{1}{20}$	4×80	=	320
$\frac{1}{20}$	6×120	=	720
$\frac{1}{20}$	8×160	=	1280
$\frac{1}{20}$	10×200	=	2000
$\frac{1}{20}$	12×240	=	2880

$\frac{1}{20} d = s$

← Student draws a correct graph showing a line that passes through the origin. Labels shown are correct. (1 point)

← Student draws an incorrect graph. (0 points)

← Student draws an incorrect graph, indicating a linear relationship between diameter and breaking strength. (0 points)

← Student does not state the correct formula. (0 points)

Overall, the student earns 1 point, demonstrating a minimal understanding of the mathematical concept of change and a minimal ability to model change graphically in two-variable situations.

INSTRUCTIONAL STRATEGIES
Grade 11 Mathematics

The open-response item **"Rope Strength"** was designed to address students' ability to (1) understand differences between linear and quadratic functions, (2) recognize the appropriate curve for a particular set of data, (3) understand how ratio can be used in a mathematical context and to solve real-world problems, (4) demonstrate how slope shows rate of change in linear functions arising from real-world situations, and (5) construct graphs and show how equations and graphs are models of the relationship between two real-world quantities. The instructional strategies below present ideas for helping students explore and master these concepts.

Review the relationships among words, tables, equations, and graphs.

Review modeling of real-world and algebraic problems with graphs.

Review linear, quadratic, absolute value, and exponential equations and graphs.

Review how to determine algebraic equations to represent relationships between two variables given graphically or in real-world problems.

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher guidance and support) any or all of the following activities:

- Practice interpreting graphs that represent a variety of real-world situations. Use a variety of graphs to explore and demonstrate understanding of how the variables relate to each other.
- Make up graphs that tell stories about real-world situations.
- Use probes and calculators or computers to collect and graph real data from investigations.
- Compare graphs of real-world situations in which the graphic representation is a line, a parabola, an exponential curve, or none of these.
- Explore tables of real-world data representing linear, quadratic, or exponential relationships and represent these relationships with an algebraic formula.
- Use problem-solving techniques such as finite differences (subtracting until finding a constant difference) to determine whether the function is linear or quadratic.